# AP Calculus - Summer Packet

## Congratulations! Welcome to Calculus!

Going into AP Calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems wrong next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra/trigonometry and not the calculus. This summer packet is intended for you to brush up and, if necessary, relearn these topics.

We assume that you have skills in Algebra and Trigonometry. Being able to solve equations, work with algebraic expressions, factor, identify trigonometric identities, know ratios for reference angles and the unit circle, for example, should now be a part of you. If not, you would not be going on to AP Calculus. So, topics we feel you absolutely need to know are included here. These are skills that are used continually in AP Calculus.

On the following pages are questions listed by topic. **Please do each problem only in the space provided. Additional pages will not be graded.** Work in pencil so you can make corrections. If you are unsure of how to attempt these problems, look at your notes, websites, or talk to friends. Building a small study group now, that can be your support group next year as well, is a good idea. Do not fake your way through these problems. Make sure you understand how to do all of these by the end. As stated, students are notoriously weak on these topics; even students whose performance has routinely been stellar can be forgetful of prerequisites, especially since skills will not be reviewed before being incorporated into the calculus.

This packet is due the first day back to school in the fall. **An assessment, in some form, will occur the first week. For example, the packet could be collected and graded. A test of representative problems could be given.** Be sure your name appears on the first sheet, all sheets are staples together, and no notebook "fringes" are left dangling. All work needs to be shown. Also, do not rely on your calculator. Remember, more than half of the AP exam is taken without a calculator. So, try and do as much as you can without a calculator – even the graphs. Remember the graphs are just sketches with intercepts, asymptotes, and end behavior being the most important.

It is a mistake to decide to complete this packet now. Let it go until mid-summer. We want these techniques to be relatively fresh in your mind in the fall. Also, do not wait until the very last minute to begin. Give yourself the last two weeks of vacation to review and complete these problems.

**BC Only: As indicated, the last few pages are required for BC students only.**

We look forward to seeing you in the next school year. ☺

AP Calculus Teachers
Stamford High School

1.) If $f(x)=x^3-2x-1$ and $g(x)=2x+5$, please find the following:

a.) $f(g(-4))$

b.) $g(f(-4))$

2.) If $f(x)=\frac{1}{x}$, please express $\frac{f(x+h)-f(x)}{h}$ as a simplified rational expression.

3.) Please rationalize the denominator of the expression $\frac{1}{\sqrt{x+h}+\sqrt{x}}$. Simplify as much as possible.

4.) Please use the *point-slope form of a line* to determine the equation of the line through the point $(7,-1)$ that is perpendicular to the line containing the points $(1,-3)$ and $(-2,5)$.

5.) Please factor the following polynomials:

a.) $4x^2-21x-18$

b.) $3x^2+6x^3-9x$

c.) $2x^3+3x^2-11x-6$

d.) $(x-1)^3(2x-3)-2(x+6)(x-1)^2$

6.) Please solve the equation $7x^4 - 42x^2 = 35x$ for all value of $x$ .

7.) Please determine the DOMAIN and the ROOTS of the following functions:

a.) $f(x) = \dfrac{x-3}{x^2-4}$

b.) $g(x) = \dfrac{\sqrt{x-2}}{x^2-x}$

c.) $h(x) = \dfrac{1}{\sqrt{x^2-4x}}$

8.) Please find the inverse of the function $f(x) = 4\sqrt[3]{x-1}$

9.) If $f(x)=x^3+Ax^2+Bx-3$ and if $f(-1)=-6$ and $f(1)=4$, what is the value of $2A+B$?

10.) Please simplify the quotient $\dfrac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$. *(Suggestion: Factor the numerator.)*

11.) Please ***use a graphing calculator*** to determine all solutions of the equation $x^3-4x^2=2x-3$. *(No work need be shown.)*

12.) Please solve the following equation for $z$ :

$$2xy^3+3x^2y^2z+4=3x^2y+x^3yz+5z$$

13.) Please solve the inequality $4x^3-21x^2-18x\leq 0$ *(No calculator, here. Sorry.)*

14.) Please ***use your graphing calculator*** to determine the values for which $3^x \geq x^4$

15.) Please find the $x$-coordinate of the points at which the curves $y-x+1=0$ and $2y+3x^2=3x+y^2$ intersect.

16.) Please rewrite $\dfrac{x^2-3x+2}{\sqrt{x}}$ as a sum of terms with rational exponents.

17.) Please simplify the following expressions completely:

a.) $\left(5x^2 \cdot x^{-2}\right)^2$

b.) $\dfrac{\left(-3x^{-3}\right)^2}{-9x^{-4}}$

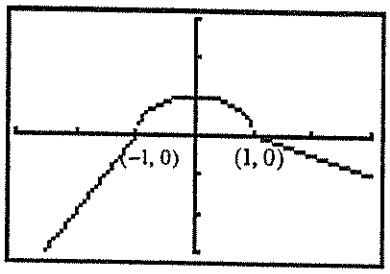
c.) $\dfrac{6x^{-2}+9x^2}{3x^{-2}}$

d.) $\dfrac{x^{-1/2}\left(2x^{1/2}-x^{-1/2}\right)}{x^{-1}}$

e.) $\left(5x^2+x^{-2}\right)^2$

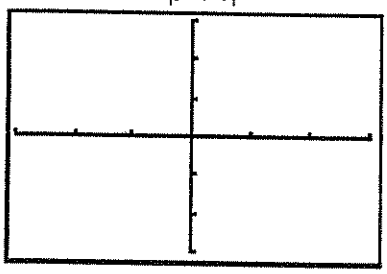
f.) $\dfrac{\left(\sqrt{2x}\right)^5}{\left(\sqrt{2x}\right)^9}$

18.) This is the graph of a function $y = f(x)$.

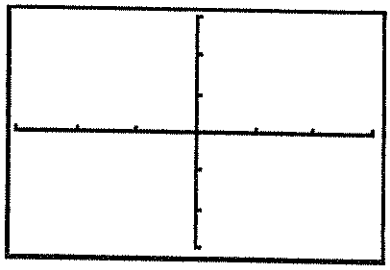



On the axes provided, please sketch the following:

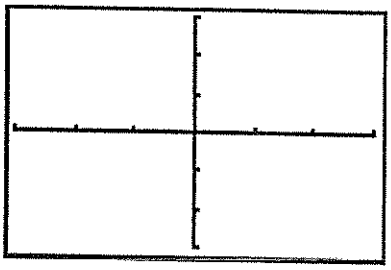
a.) $y = |f(x)|$



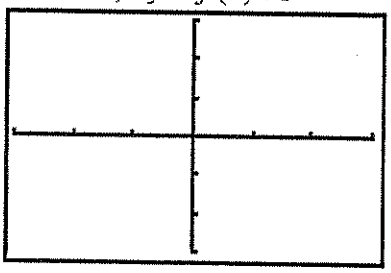
b.) $y = f(x+1)$



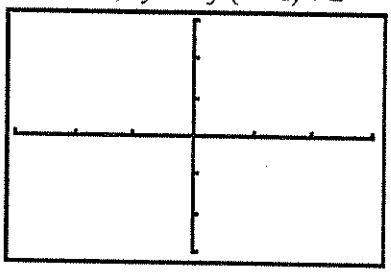
c.) $y = -2f(x)$



d.) $y = f(x)+1$



e.) $y = -f(x-1)+2$

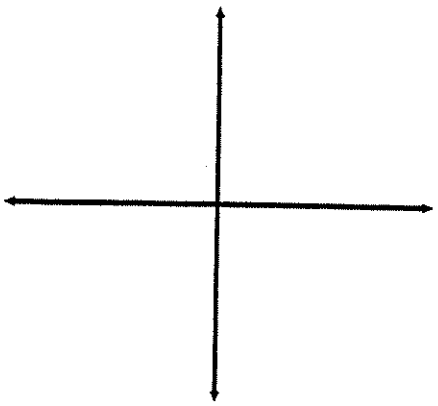


19.) a.) What are the domain and range of the function $f(x) = 4 - x^2$ ?

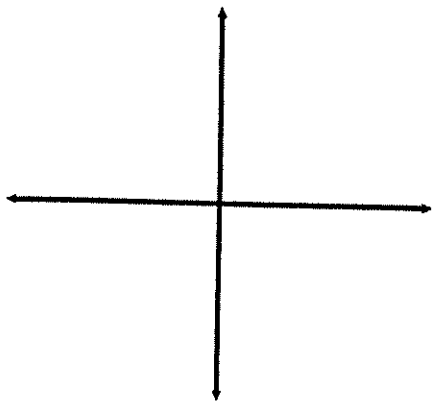
b.) What are the domain and range of the function $f(x) = 4 - x^2$ , where $1 \le x < 3$ ?

20.) On the axes provided, please sketch the graph of the following:

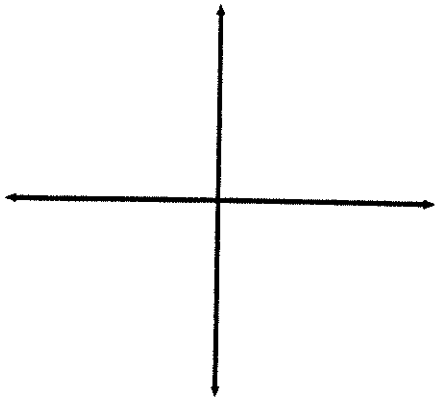
a.) $y=-x^2+4x-4$



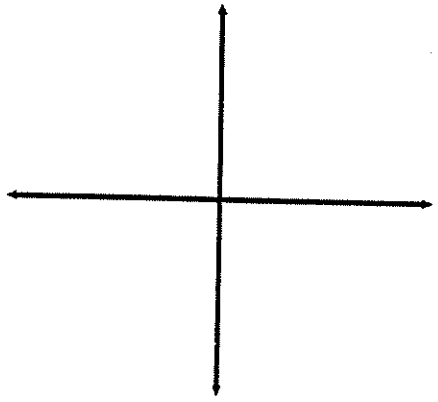
b.) $y=\begin{cases}2x, & \text{if } x<-1\\ 2x^2+x-3, & \text{if } -1\le x<2\\ -x+3, & \text{if } x\ge 2\end{cases}$



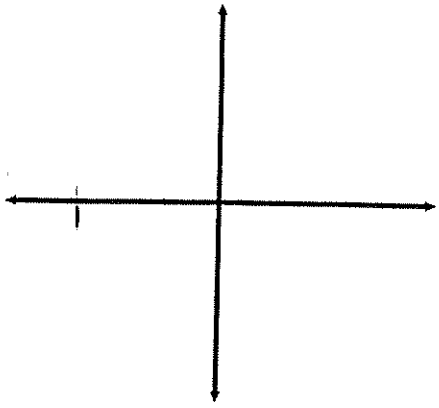
c.) $y=\sqrt{36-x^2}$



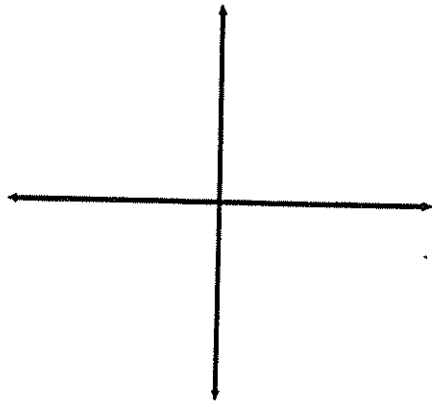
d.) $5x^2+8y^2=80$



e.) $y=x^{2/3}$



f.) $y=a^x$, where $a>1$



21.) Please solve each of the following for $x$. Identify any domain restrictions. *(Note: You may need to leave your answer in logarithmic or exponential form)*

a.) $\log(x-2)=3$

b.) $9^x=4$

c.) $\ln x+\ln(2x+5)=\ln 7$

d.) $2e^{2-x}+5=12$

22.) Please simplify $\log_2 5-2\log_2\left(\frac{1}{2}\right)+\log_2 1.6$ so that there are no logarithms in your answer.

23.) Please write the expression $\log_b\left(\frac{(4x^5-x-1)\sqrt{x-7}}{\sqrt[3]{x^2+1}}\right)$ as a sum of logarithms.

24.) Given a box of height $h$ and whose base dimensions are $x$ and $y$, please express the *surface area AND the volume* of the box, given the following conditions:

a.) The box is closed on all sides

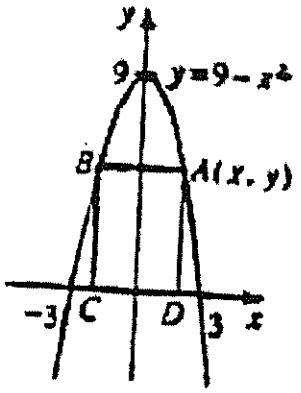
b.) The box has an open top

c.) The box has an open top and a square base.

25.) A 6-foot tall person is standing $x$ feet away from a 10 foot lamppost. What is the distance $d$ from the base of the lamppost to the end of the person's shadow, expressed as a function of $x$?

26.) A piece of wire 5 meters long is to be cut into two pieces. One piece is $x$ inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Please find an expression for the total area made up by the square and the circle as a function of $x$.

As shown to the right, rectangle ABCD has vertices C and D on the x-axis and vertices A and B on the part of the parabola $y = 9 - x^2$ that is above the x-axis.

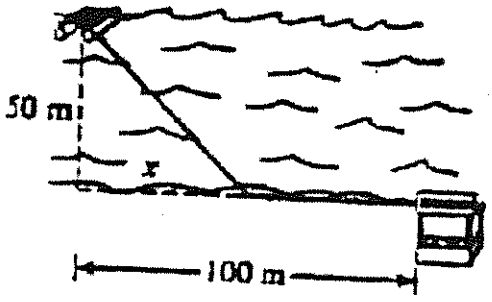



a. Express the perimeter of the rectangle, *P*, as a function of the x-coordinate of A.

b. What is the domain of the perimeter function?

c. For what value of x is the perimeter a maximum?

From a raft 50 m offshore, a lifeguard wants to swim to shore and run to a snack bar 100 m down the beach, as shown to the right.




a. If the lifeguard swims at 1 m/s and runs at 3 m/s, express the total swimming and running time, *t*, as a function of the distance x shown in the diagram.

b. Use a computer or graphing calculator to find the minimum time.

27.) Please determine the following (Remember . . . . No calculator!)

a.) $\cos 0$ b.) $\sin 0$ c.) $\tan\frac{\pi}{2}$ d.) $\cos\frac{5\pi}{6}$

e.) $\sec\frac{7\pi}{4}$ f.) $\tan 660^\circ$ g.) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ h.) $\tan^{-1}(-1)$

28.) Given the function $f(x)=2\sin\left(\frac{2}{3}x\right)+1$ , please determine the

a.) amplitude b.) range c.) period

29.) Please circle the expressions that are identical:

$\cos^2 x$ $(\cos x)^2$ $\cos\left(x^2\right)$ $1+\sin^2 x$

30.) Please circle the expressions that are identical:

$(\sin x)^{-1}$ $\sin^{-1} x$ $\sin\left(x^{-1}\right)$ $\frac{1}{\sin x}$

31.) Please solve each of the following for $\theta \in [0, 360^\circ)$

a.) $\tan^2\theta - 3 = 0$

b.) $\tan\theta - \cot\theta = 0$

c.) $\cos 2\theta = -\frac{1}{2}$

d.) $4(1-\sin\theta)(1+\sin\theta) = 3$

## Complex Fractions

Simplify the following:

1. $\dfrac{x}{x-\frac{1}{2}}$

2. $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4. $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5. $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

6. $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

7. $\dfrac{x^{-3}-x}{x^{-2}-1}$

8. $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

9. $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$

## Asymptotes

For each function, find the equation of both the vertical asymptote(s) and horizontal asymptote(s) if they exist

1. $y=\frac{x}{x-3}$

2. $y=\frac{x+4}{x^2-1}$

3. $y=\frac{x+4}{x^2+1}$

4. $y=\frac{x^2-2x+1}{x^2-3x-4}$

5. $y=\frac{x^2-9}{x^3+3x^2-18x}$

6. $y=\frac{2x^2+6x}{x^3-3x^2-4x}$

7. $y=\frac{x^2-x-6}{x^3-x^2+x-6}$

8. $y=\frac{2x^3}{x^3-1}$

9. $y=\frac{\sqrt{x}}{2x^2-10}$

Simplify

1. $\frac{\sqrt{x}}{x}$

2. $e^{\ln 3}$

3. $e^{3\ln x}$

4. $e^{1+\ln x}$

5. $\ln 1$

6. $27^{\frac{2}{3}}$

7. $\ln e^{7}$

8. $\log_3\left(\frac{1}{3}\right)$

9. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

10. $\log_{\frac{1}{2}} 8$

11. $\ln\left(\frac{1}{2}\right)$

The following additional pages are for students taking AP Calculus BC.

BC only

## Sequences and Series

1. Write the first 5 terms of the sequence

   a. $a_n = \frac{n}{n+1}$

   b. $a_n = \sin\frac{n\pi}{2}$

   c. $a_n = (-1)^n\left(5 - \frac{1}{n} + \frac{1}{n^2}\right)$

   d. $a_n = \frac{3n!}{(n-1)!}$

2. Write an expression for the nth term of the sequence

   a. $1,4,7,10.....$

   b. $\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6}....$

   c. $2,-1,\frac{1}{2},\frac{-1}{4},\frac{1}{8}....$

   d. $1,2,7,14,23.....$

   e. $\frac{2}{1},\frac{4}{3},\frac{8}{5},\frac{16}{7},\frac{32}{9}....$

   f. $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}....$

   g. $\frac{1}{2},\frac{\sqrt{2}}{3},\frac{\sqrt[3]{3}}{4},\frac{\sqrt[4]{4}}{5}....$

   h. $1.0,-1.0,1.0,-1.0.....$

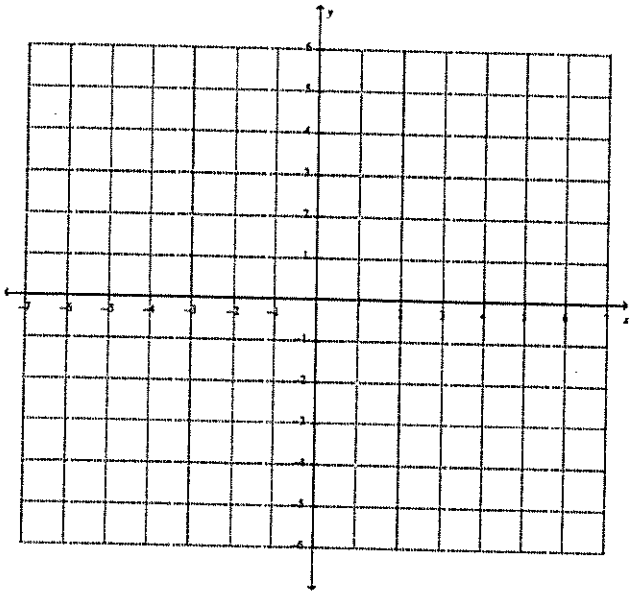
   i. $\frac{-2}{1},\frac{8}{2},\frac{-26}{6},\frac{80}{24},\frac{-242}{120}....$

**: Parametrics**

1. Sketch the curve described by the parametric equations

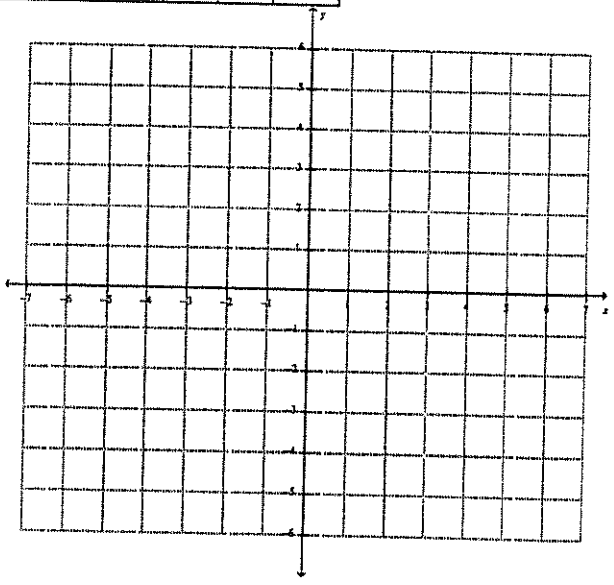
a. $x = t^2 - 4$ *and* $y = \frac{t}{2}$ *where* $-2 \le t \le 3$

| $t$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $x$ | | | | | | |
| $y$ | | | | | | |



b. $x = 4t^2 - 4$ *and* $y = t$ *where* $-1 \le t \le \frac{3}{2}$

| $t$ | -1 | -.5 | 0 | .5 | 1 | 1.5 |
|---|---|---|---|---|---|---|
| $x$ | | | | | | |
| $y$ | | | | | | |



2. Eliminate the parameter in the following parametric equations. In each problem, it will usually be easier to solve for t in one equation than the other.

   a. $x = t - 3$ *and* $y = t^2 + \sqrt{t} - 2$

   b. $x = 3t + 2$ *and* $y = \dfrac{1}{2t - 1}$

   c. $x = \dfrac{t}{2}$ *and* $y = \sin(t + 1) - 1$

   d. $x = \dfrac{1}{\sqrt{t+1}}$ *and* $y = \dfrac{t}{t+1}$ *when* $t > -1$

## : Polars

1. For each polar point. Label it in two other ways.
   a. $(4,60^\circ)$

   b. $(-5,315^\circ)$

   c. $(2,-90^\circ)$

   d. $\left(1,\frac{5\pi}{6}\right)$

   e. $\left(-8,\frac{\pi}{6}\right)$

   f. $\left(\frac{-3}{2},\frac{-5\pi}{3}\right)$

2. Convert the following polar points to rectangular coordinates
   a. $(6,90^\circ)$

   b. $(4,60^\circ)$

   c. $(10,225^\circ)$

   d. $(5,\pi)$

   e. $\left(2\sqrt{3},\frac{\pi}{6}\right)$

   f. $\left(\frac{5}{2},\frac{5\pi}{3}\right)$

3. Convert the following rectangular points to polar coordinates
   a. $(-5,-5)$

   b. $(0,-2)$

   c. $(1,-\sqrt{3})$

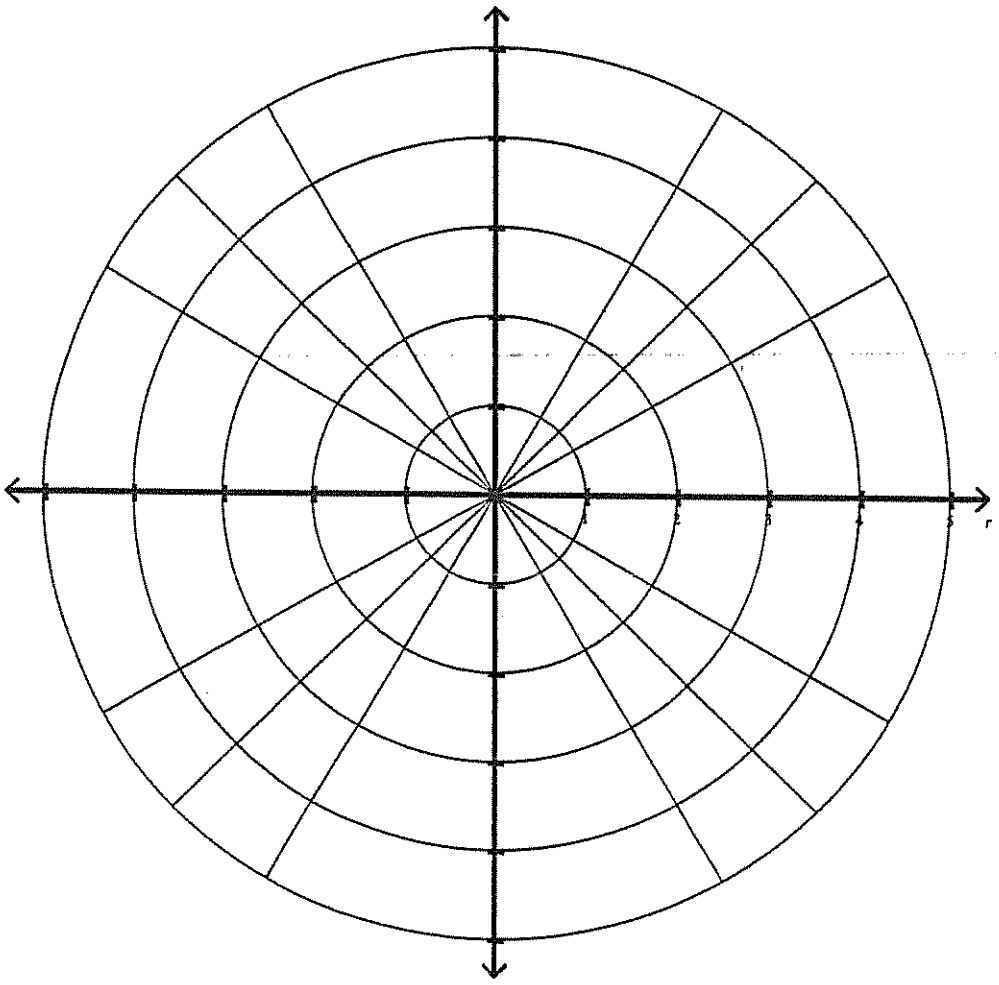
   d. $(-7,0)$

   e. $(5,12)$

   f. $(6,-3)$

4. Plot the points(1 decimal place) and sketch the graph of the polar equation $r = 3 + 2\sin\theta$

| $\theta$ | 0° | 30° | 60° | 90° | 120° | 150° | 180° | 210° | 240° | 270° | 300° | 330° | 360° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r$ | | | | | | | | | | | | | |



5. Match the polar equations with the graphs below.

a. $r = 3 - \cos\theta$

b. $r = 2 - 2\sin\theta$

c. $r = 5\cos 3\theta$

d. $r = 2 + 2c\cos\theta$

e. $r = 3 + 1.5\sin\theta$

f. $r = 3.5\cos 2\theta$

g. $r = 5\sin 3\theta$

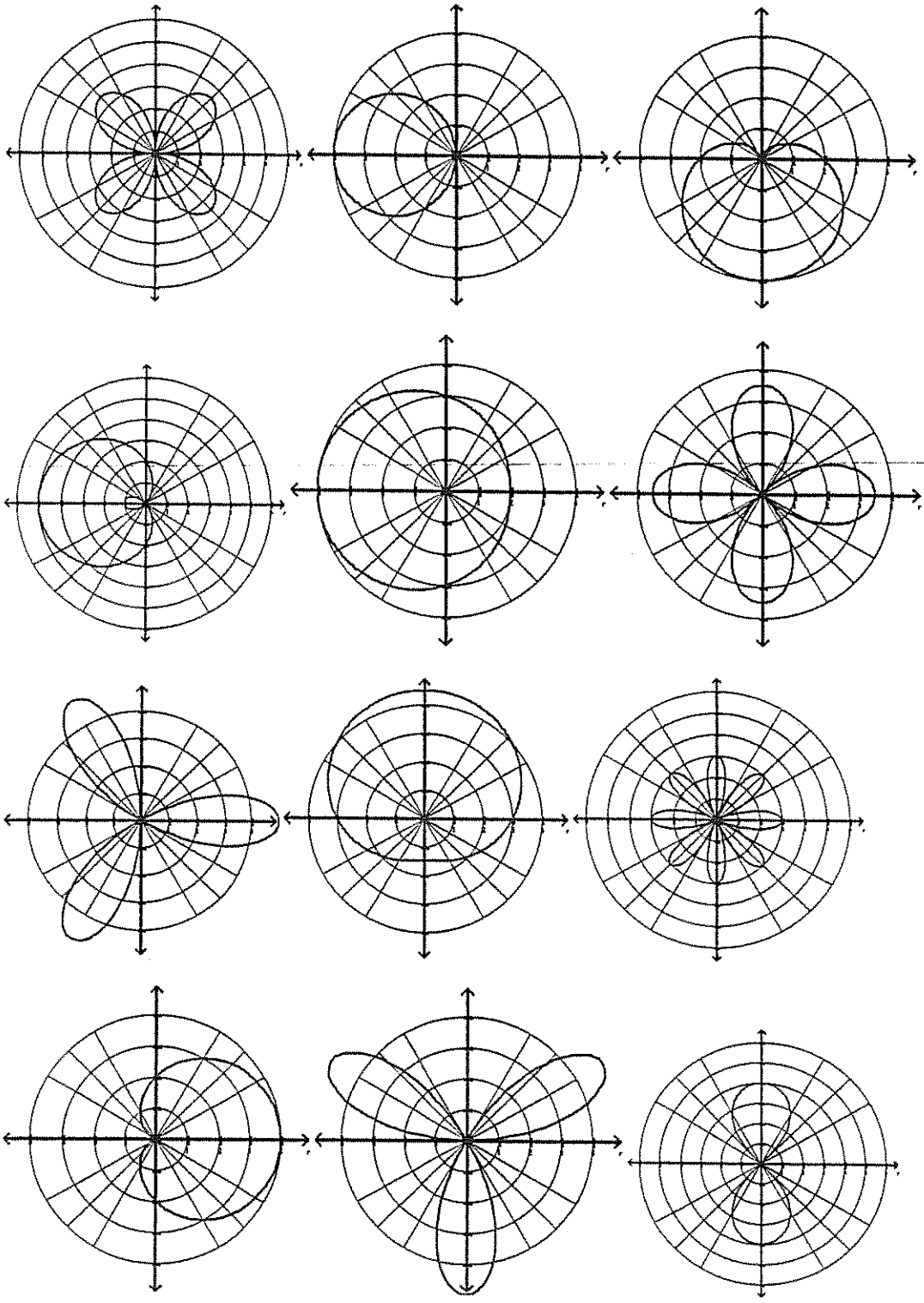
h. $r^2 = -16\cos 2\theta$

i. $r = 2 - 3\cos\theta$

j. $r = 3\cos 4\theta$

k. $r = -4\cos\theta$

l. $r = 3.5\sin 2\theta$



## Limits

1. $\lim_{x\to 0} \frac{2}{1-e^{\frac{1}{x}}}$

2. $\lim_{x\to 0} \sqrt{2+\arctan(\frac{1}{x})}$

3. $\lim_{x\to -\infty} \frac{-7x}{\sqrt{4x^2+3}}$

4. $\lim_{x\to \infty} \sqrt{x^2+x+2}-x$

5. $\lim_{x\to 0} \frac{x^2-16}{x^2+2x-24}$

6. $\lim_{x\to -1} \frac{x^2-4x-5}{x+1}$

7. $\lim_{x\to -3^-} \sqrt[3]{x+3}\ln(x+3)$

8. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

9. $\lim_{x\to -\infty} \frac{x^2-2x-3}{-5x^2}$

10. $\lim_{x\to -\infty} \frac{x^4+2x^2-3x}{4x^3}$

11. $\lim_{x\to -\infty} \frac{x-1}{x+2}$

12. $\lim_{x\to 2^-} \frac{x^2-x-6}{x-2}$

13. Find the equation of the tangent line for $f(x)=\frac{1}{2x+3}$ by finding the derivative, at $x=2$

14. $f(x)=x^2+2x+3$. Find $f'(x)$. Then evaluate $f'(1)$